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Inner bremsstrahlung in electron pair creation in general nuclear transitions

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Abstract. The description of these processes is extended to general spin transitions. The decay distributions, both for pair creation and its inner bremsstrahlung, are given for experiments where no polarizations are observed, in terms of two structure functions which may be measured in either process; in most simple cases where a single multipole dominates these are theoretically related.

We here extend the description of zero–zero transitions (Burkhardt *et al* 1974, to be referred to as I) to electromagnetic transitions between nuclear states of arbitrary spin. We give decay distributions for pair creation P and its inner bremsstrahlung B in experiments where no polarizations are measured either for the nuclear states or for the leptons, relating these to the direct radiation rate R. The treatment of the lepton final states is similar to the zero–zero case and we simply quote the more general results.

The nuclear transition current can, however, have a more complex structure; we use the results of Durand *et al* (1962), relating the Lorentz covariant current matrix elements to the usual multipoles. The spin averaged decay angular distribution depends on the nuclear states through just two structure functions, which in turn depend only on the magnitude of the recoil momentum $q_3^2 = \Delta^2 - q^2$. These may be measured from R and P, among other possibilities, so that B can be predicted.

The matrix element for each process has the form

$$\mathcal{M} = j_\mu l^\mu \tag{1}$$

where the lepton transition potential l^μ has the familiar forms

$$r_\pi^\mu = \epsilon_\pi^\mu \tag{2a}$$

$$p_{ts}^\mu = -\frac{e}{q^2}(\bar{u}_t(p_-)\gamma^\mu v_s(p_+)) \tag{2b}$$

$$b_{ts\pi}^\mu = -\frac{e^2}{q^2}\left(\bar{u}_t(p_-)\left(\epsilon_\pi\frac{(\not{p}_- + \not{k} + m)}{2p_- \,.\, k}\gamma^\mu - \gamma^\mu\frac{(\not{p}_+ + \not{k} - m)}{2p_+ \,.\, k}\epsilon_\pi\right)v_s(p_+)\right) \tag{2c}$$

for the three processes. The nuclear, transition current j^μ, which is independent of the process, may be written in terms of spin states quantized in the recoil direction:

$$j^\mu \equiv \langle P'S'\lambda'|J^\mu|PS\lambda\rangle = \Gamma^\mu_{\lambda'\lambda}(q_3) \tag{3}$$

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$\boldsymbol{P}$, S and λ are the initial nuclear momentum, spin and helicity, the primed quantities describing the final state. In the usual approximation $\Delta/M \ll 1$, the current components are the same in the laboratory ($\boldsymbol{P} = 0$) and brickwall ($\boldsymbol{P} = -\boldsymbol{P}'$) frames and we shall work in the former.

If the initial nuclear state is unpolarized and no spin measurements are made in the experiment, the decay distribution

$$\mathrm{d}\lambda = \frac{1}{2S+1}\sum_{\text{spins}}\frac{1}{2M}|\mathscr{M}|^2\frac{\mathrm{d}\Phi}{2M} \tag{4}$$

where $\mathrm{d}\Phi$ is the appropriate element of lepton phase space; it may be written

$$\mathrm{d}\lambda = W^{\mu\nu}L_{\mu\nu}\,\mathrm{d}\Phi \tag{5}$$

where

$$W^{\mu\nu} = \frac{1}{2S+1}\sum_{\lambda\lambda'}\frac{1}{4M^2}\Gamma^{\mu}_{\lambda'\lambda}\Gamma^{*\nu}_{\lambda'\lambda}, \qquad L^{\mu\nu} = \sum_{\substack{\text{spins}\\ \text{polarizations}}} l^{\mu}l^{*\nu}. \tag{6}$$

For those who do not wish to follow the derivation, the angular distributions (5) for each process are obtained, in terms of two nuclear structure functions $G^{\mathrm{T}}(q_3^2)$ and $G^{\mathrm{L}}(q_3^2)$, by using (17) and (18); the lepton factors L_{00} and L^{μ}_{μ} in (18) are given for R, P and B by (19), with (20) and (I. 13). G^{T} and G^{L} are related to the usual multipole matrix elements (Willey 1963, particularly equations (2.17) and (2.32))† by (12).

We first examine in detail the nuclear structure tensor $W^{\mu\nu}$. Lorentz and gauge invariance require that

$$W^{\mu\nu} = M_1(q^2)[q^2g^{\mu\nu}-q^{\mu}q^{\nu}]-\frac{M_2(q^2)}{M^2}[q^2P^{\mu}P^{\nu}-q\,.\,P\{q,P\}^{\mu\nu}+(q\,.\,P)^2g^{\mu\nu}] \tag{7}$$

where

$$\{a,b\}^{\mu\nu} \equiv a^{\mu}b^{\nu}+a^{\nu}b^{\mu}.$$

The scalar structure functions M_1 and M_2 are related to the current matrix elements by

$$M_1 = \frac{G^{\mathrm{T}}-G^{\mathrm{L}}}{q_3^2}, \qquad M_2 = \frac{q_0^2G^{\mathrm{T}}-q^2G^{\mathrm{L}}}{q_3^2q_0^2} \tag{8}$$

where

$$G^{\mathrm{T,L}} \equiv \frac{1}{2S+1}\sum_{\lambda'\lambda}\frac{1}{4M^2}|\Gamma^{\mathrm{T,L}}_{\lambda'\lambda}|^2 \tag{9}$$

and $\Gamma^{\mathrm{T}} \equiv \Gamma^{(+)}$ and $\Gamma^{\mathrm{L}} \equiv \Gamma^3$ are the transverse and longitudinal components (Durand *et al* 1962).

The current matrix elements are related to the usual multipoles by (Willey 1963)

$$\sqrt{2}\Gamma^{(\pm)}_{\lambda'\lambda} = \mp\Gamma^1_{\lambda'\lambda}-\mathrm{i}\Gamma^2_{\lambda'\lambda} = (-1)^{2S'}\sum_{J=1}^{\infty}\begin{pmatrix}S' & J & S\\ \lambda' & \pm 1 & \lambda\end{pmatrix}[X^J_+E_J \pm X^J_-M_J] \tag{10a}$$

† Our three multipoles $-(J+1/J)^{1/2}Q_J$, E_J and $\mathrm{i}M_J$ are equal to his $\langle J_{\mathrm{f}}\|M(\sigma,q_3)\|J_{\mathrm{i}}\rangle N_J$, where $\sigma = CJ$, EJ and MJ, and

$$N_J = -\frac{32\pi^2M\mathrm{i}^Jq_3^J}{2J+1!!}\left[\left(\frac{J+1}{J}\right)\left(\frac{2J+1}{4\pi}\right)\right]^{1/2}.$$

and

$$\frac{q_3}{q_0}\Gamma^3_{\lambda'\lambda} = \Gamma^0_{\lambda'\lambda} = (-1)^{2S'} \sum_{J=0}^{\infty} \begin{pmatrix} S' & J & S \\ \lambda' & 0 & \lambda \end{pmatrix} X^J_+ Q_J \tag{10b}$$

where $X^J_\pm \equiv \frac{1}{2}[1 \pm (-1)^{J+\pi}]$ project natural and unnatural parity transitions respectively, π being the relative parity of the two states. The threshold behaviour of the scalar, electric and magnetic multipole form factors is normally in nuclei of the form

$$Q_J \sim q_3^J, \qquad E_J \sim q_3^J, \qquad M_J \sim q_3^J\left(\frac{1}{M_pR}\right) \tag{11}$$

indicating the contributions that will dominate in any particular case. Using the orthogonality of the Clebsch–Gordan coefficients, the structure functions (9) are given by

$$\begin{aligned}(2S+1)4M^2G^{\mathrm{T}} &= \sum_{J=1}^{\infty} \frac{1}{2J+1}(X^J_+|E_J|^2 + X^J_-|M_J|^2) \\ (2S+1)4M^2G^{\mathrm{L}} &= \frac{q_0^2}{q_3^2}\sum_{J=0}^{\infty}\frac{1}{2J+1}X^J_+|Q_J|^2.\end{aligned} \tag{12}$$

The important case of the zero–zero transition, treated directly in I, is exceptional since the dominant monopole

$$Q_0 = e2Ma_0q_3^2 + \mathrm{O}(q_3^4), \tag{13}$$

the constant, q_3^0, term vanishing from the orthogonality of the two states. With $G^{\mathrm{T}} = 0$ and $G^{\mathrm{L}} = e^2a_0^2q_0^2q_3^2$, $W^{\mu\nu}$ reduces to

$$W^{\mu\nu} = \frac{e^2a_0^2}{4M^2}4[q^2P^\mu - (q\,.\,P)q^\mu][q^2P^\nu - (q\,.\,P)q^\nu] \tag{14}$$

corresponding to that formed, using (3), and (6) from the current (6) in I.

The calculation of the lepton tensor $L^{\mu\nu}$ for the three processes follows the standard techniques for spin and polarization sums, which yield

$$R^{\mu\nu} = \sum_{\pi=\pm} \epsilon^\mu_\pi\epsilon^{*\nu}_\pi = -g^{\mu\nu} \tag{15a}$$

$$P^{\mu\nu} = \sum_{t,s} p^\mu_{ts}p^{*\nu}_{ts} = \frac{2e^2}{(q^2)^2}(-q^2g^{\mu\nu} + 2\{p_+, p_-\}^{\mu\nu}) \tag{15b}$$

$$B^{\mu\nu} = \sum_{t,s,\pi} b^\mu_{ts\pi}b^{*\nu}_{ts\pi} = \frac{4e^4}{(q^2)^2}(B^{\mu\nu}_{+-} + B^{\mu\nu}_{++} + B^{\mu\nu}_{--} + B^{\mu\nu}_{+} + B^{\mu\nu}_{-}) \tag{15c}$$

where

$$\begin{aligned} B^{\mu\nu}_{+-} &= \frac{p_+\,.\,p_-[2\{p_+, p_-\}^{\mu\nu} + \{p_+ + p_-, k\}^{\mu\nu} - 2(p_+\,.\,p_- + m^2)g^{\mu\nu}] - 2m^2k^\mu k^\nu}{(p_+\,.\,k)(p_-\,.\,k)} \\ B^{\mu\nu}_{++} &= \frac{m^2}{(p_+\,.\,k)^2}[g^{\mu\nu}(p_-\,.\,k + p_+\,.\,p_- + m^2) - \{p_+, p_-\}^{\mu\nu} - \{p_-, k\}^{\mu\nu}] \\ B^{\mu\nu}_{+} &= \frac{g^{\mu\nu}(m^2 - p_-\,.\,k - 2p_+\,.\,p_-) + \{p_+, p_-\}^{\mu\nu} + \{p_-, k\}^{\mu\nu} - 2p^\mu_+p^\nu_+}{(p_+\,.\,k)}.\end{aligned} \tag{16}$$

The $L^{\mu\nu}$ are manifestly hermitian and, from time reversal, are real. In the normalization, $\bar{u}u = 2m$, the lepton phase space elements are

$$d\Phi_R = \frac{d^3k}{2k}\frac{\delta(k-\Delta)}{(2\pi)^2} = \frac{\Delta}{2\pi}\frac{d\Omega}{4\pi} \tag{17a}$$

$$d\Phi_P = \frac{d^3p_+}{2E_+}\frac{d^3p_-}{2E_-}\frac{\delta(E_++E_--\Delta)}{(2\pi)^5} = \frac{p_+p_-}{2(2\pi)^3}\,dE_+\,dE_-\,dz\,\delta(E_++E_--\Delta) \tag{17b}$$

$$\begin{aligned} d\Phi_B &= \frac{d^3k}{2k}\frac{d^3p_+}{2E_+}\frac{d^3p_-}{2E_-}\frac{\delta(E_++E_-+k-\Delta)}{(2\pi)^8} \\ &= \frac{p_+p_-k}{4(2\pi)^6}\,dE_+\,dE_-\,dk\,dz_+\,dz_-\,d\phi_{+-}\,\delta(E_++E_-+k-\Delta). \end{aligned} \tag{17c}$$

In evaluating $W^{\mu\nu}L_{\mu\nu}$, the terms in $W^{\mu\nu}$ involving q^μ or q^ν give no contribution since $q^\mu L_{\mu\nu} = 0$ from current conservation (P, B) or transversality (R). We obtain

$$W^{\mu\nu}L_{\mu\nu} = L^\mu_\mu(q^2M_1 - q_0^2M_2) - L_{00}q^2M_2 = G^{\mathrm{T}}\left(\frac{q^2}{q_3^2}L_{00} - L^\mu_\mu\right) + G^{\mathrm{L}}\frac{(q^2)^2L_{00}}{q_3^2q_0^2}. \tag{18}$$

Since for R, $q^2 = 0$, (15a) and (18) give

$$W^{\mu\nu}R_{\mu\nu} = 2G^{\mathrm{T}}(q^2 = 0) \tag{19a}$$

while for P and B (15) gives

$$P^\mu_\mu = \frac{-4e^2}{(q^2)^2}(q^2+2m^2), \qquad P_{00} = \frac{2e^2}{(q^2)^2}(4E_+E_- - q^2) \tag{19b}$$

$$B^\mu_\mu = \frac{4e^4}{(q^2)^2}S, \qquad B_{00} = \frac{4e^4}{(q^2)^2}T \tag{19c}$$

where the expression for T is (I.13) and S, in the same notation, is given by

$$\begin{aligned} S_{+-} &= \frac{4p_+\cdot p_-}{(p_+\cdot k)(p_-\cdot k)}(p_+\cdot p_- + 2m^2) \\ S_{++} &= \frac{-2m^2}{(p_+\cdot k)^2}(p_+\cdot p_- + p_-\cdot k + 2m^2) \\ S_{+} &= \frac{2}{p_+\cdot k}(2p_+\cdot p_- + p_-\cdot k - m^2). \end{aligned} \tag{20}$$

We shall not attempt to give integrated spectra here, since they depend on the multipoles involved so that the variety of possibilities is large, while the experimental configuration often limits angular acceptance. Integrations over the full angular range can be evaluated in terms of standard functions, though they are more complicated than in the zero–zero case because a third different denominator factor, q^2, enters. The apparent singularity at $q_3^2 = 0$ in (18) is not real since lepton current conservation requires

$$q_0^2L_{00} = q_0q_3L_{03} = q_3^2L_{33}$$

and none of the $L_{\mu\nu}$ is singular. It is nonetheless a point to watch in numerical evaluation.

Measurement of the distribution of the pair creation process P in two independent variables, such as E_+ and q^2, is sufficient to determine $G^L(q^2)$ and $G^T(q^2)$ in the relevant region, $4m^2 < q^2 < \Delta^2$; the bremsstrahlung distribution is then completely predicted.

We have preserved the distinction between scalar (Coulomb) and electric multipoles but when, as in most decays, $q_3R \ll 1$, these are related (Willey 1963) by

$$E_J = \left(\frac{J+1}{J}\right)^{1/2} Q_J[1+O(q_3^2R^2)] \tag{21}$$

so their different structures would only be significant in transitions where the leading electric multipoles were anomalously small. It follows from (12) and (21) that in simple situations, where a single multipole dominates, G^L and G^T are related, so a simple measurement of the q^2 dependence is sufficient to determine the multipole and, if the relative parity is known, the bremsstrahlung spectrum.

These results ignore the Coulomb interaction between the nucleus and the outgoing electrons, which will be important for low energy electrons or heavy nuclei.

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